Unpacking Student Conceptions of Surface Area to Volume Ratio in the Nanoscience Context:
An Empirical Application of the Construct-Centered Design Framework

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Abstract: *"Surface area to volume ratio" has been widely acknowledged as one of the "big ideas" of nanoscience, as it lays the foundation for understanding size-dependent properties that characterize nanoscale science and technology. Though seemingly an easy concept, students are reported to have difficulty grasping this concept, particularly its connection to property change. We report in this paper our effort in unpacking students' conceptions of this concept in the context of an undergraduate engineering course. Guided by the "Construct-Centered Design" (CCD) framework, we conducted detailed unpacking of student conceptions, and developed corresponding assessment items in an iterative process, which not only revealed an interesting range of conceptions, but also yielded several effective assessment items. The identified conceptions are summarized in a preliminary typology, which includes three major types of conceptions distinguished by seven aspects of variation. Practical implications of the typology and the use of the CCD process are discussed.*

Introduction

Identified as one of the "big ideas" (Stevens, Sutherland, Schank, & Krajcik, 2007) and a potential threshold concept (Park & Light, in press) of nanoscale science, "surface area to volume ratio" (SA/V) explains many properties (such as reactivity and melting point) that exhibit different behaviors at the nanoscale, a unique feature that characterizes nanoscale science and technology. As such, a sophisticated understanding of SA/V can be regarded as the prerequisite to learning size-dependent properties and other advanced nanoscience concepts.

Though the idea may seem as simple as the division between an object's surface area (SA) and volume (V), students often have difficulty grasping its meaning. Studies have shown that many European high school students (around 50%) held the incorrect intuition that objects having the same shape (e.g. cube) have the same SA/V (Tirosh & Stavy, 1999; Van Dooren, De Bock, Weyers, and Verschaffel, 2004). Even elementary school teachers have trouble truly understanding the concepts beyond simply using the mathematical equation (Cohen et al., 1999). This concept is often taught and assessed without access to students' previous level of understanding.

The same problem exists at the undergraduate level. Undergraduate students are often assumed to have mastered the concept of SA/V, but when asked to explain the connection between SA/V and property change, many students who could easily do the mathematical calculation failed to provide a sophisticated answer (Light, Swarat, Park & Drane, 2008). Using a task-based, think-aloud interview, our initial effort to unpack students' conceptions of this concept revealed that many students understand the relationship between SA/V and property change only in terms of SA, and ignore the role of V and thus miss the key idea that it is the *ratio*, not the independent variables of SA or V that determine size-dependent properties.

The study reported here continues this unpacking effort, but in addition, describes our attempt at developing assessment items to identify the variation within students' conceptions of SA/V. This process is informed by the Construct-Centered Design (CCD) framework (Shin, Stevens, Pellegrino, Krajcik, & Geier, 2008), which suggests a three-step iterative cycle to align learning goals with performance assessment: 1) Unpacking the concept; 2) Creating a claim that clarifies what students should know; 3) Identifying tasks and evidence that demonstrate that students have satisfied the claim. In our case, we developed several assessment items based on the results of the aforementioned study (Light et al., 2008), the results of which in turn revealed additional ways to unpack students' conceptions of SA/V. A typology describing the variation among student conceptions is established based on the study results.

Method

Assessment items

Five assessment items (Appendix 1) were developed to target the variation within students' understanding of SA/V based on the findings of the previous study (Light et al., 2008). Specifically, they focus on whether and how students conceptualize the connections between SA/V and its components (i.e. SA, V) and property change. The items are situated in different disciplines to see if students' understanding is limited to certain context. All items have been previously tested with small groups of students at various levels, including high school students and non-engineering majors. Students seemed to be able to answer the questions despite differences in their disciplinary knowledge. All items consisted of two parts – multiple-choice and written justification for the choice. We required students to offer explanations for their multiple-choice answers in the hope that we could get a glimpse into their thinking.

Participants and procedures

Sixteen students enrolled in a freshman engineering design course (EDC) at a major US research university participated in the study. The EDC course had a special content focus on nanoscience, and the main project of the course was to design curricular modules on basic nanoscience concepts (e.g. size and scale, surface area to volume ratio) for middle school students. In addition to design skills, students were also expected to learn about the nano-concepts through the design experience. Most of the students were engineering majors, and had a relatively solid background in math and science (e.g. taking Advanced Placement courses in math and science in high school).

The SA/V assessment items were administered to the students as part of a longer survey during regular class periods at the beginning (pre-survey) and the end (post-survey) of the course. The items were not presented in sequence in order to minimize the chance that a strategic student would find the common theme and use a generic answer (i.e. SA/V) for all items. Students answered the items independently in approximately 15-20 minutes. Fifteen students completed both the pre- and post-survey, and one student only completed the post-survey, which resulted in 31 sets of responses.

Data analysis

The multiple-choice part and the written justification part for each item were reviewed holistically, as it is our belief that a correct multiple-choice answer does not necessarily reflect sophisticated underlying reasoning. As the items were designed to tap into specific aspects of students' understanding of SA/V, the responses were not analyzed using codes generated inductively. Instead, for each item, the responses were coded for the components of SA/V that students connected with property change – surface area, volume, the ratio between the two, surface molecules/atoms, or characteristics of surface molecules/atoms. That is, each incidence in which students referred to any of the components was labeled according to the component it corresponded to.

Each item was first coded individually, and coding outcome of items exploring the same issue was then combined to get a general view of students' understanding. Students' pre- and post-survey responses were reviewed as independent entries, and then comparisons were made to explore whether there were any pre vs. post differences.

Results

As expected, students employed five types of SA/V-related explanations for property change as object's size decreases: increased surface area exposure, increased surface molecule/atom exposure, decreased volume, higher ratio between SA and V, and the different characteristics or behaviours of surface molecules/atoms. Judging from the total number of coded incidences (Figure 1), students' reasoning involving the SA aspect of SA/V (n=97) was much more frequent than the other aspects, a pattern that was consistent across all items and in both pre- and post-surveys. In many students' case, SA was the focus of their explanations for all items:

- Jena wrote: "More folds = more surface area = more reactivity" (Q6), "Smaller = more surface area = more reactivity" (Q7 and 9), and "More surface area = more reactivity; smaller tomatoes have more surface area" (Q10).
- Timothy wrote: "Surface area is the largest factor in this situation in determining reaction rate" (Q4), "Greater surface area = faster reaction time" (Q7), and simply "Surface area" (Q6, 9, and 10).

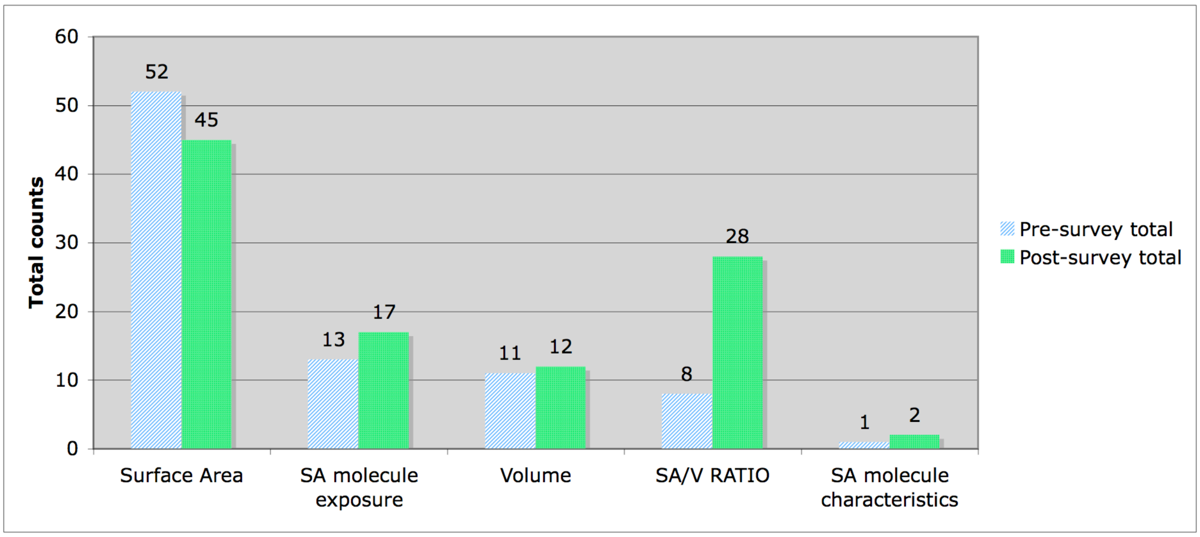

Figure 1. Total counts of coded incidences for different components of SA/V ratio in pre- and post-survey (Q 4, 6, 7, 9, and 10)

In contrast to SA, the V aspect of SA/V was mentioned much less (n=33), and most occurrences were seen in Q9 and 10 perhaps because the relevance to volume was more pronounced due to the wording or context of these items. For instance, Keith wrote in Q9: "I think the rate of reaction depends primarily on the amount of material available to react." Kyle responded in Q10: "It'll take less time to get to the center of the smaller tomatoes". And Richard answered similarly in Q10: "(Cherry tomato has) Less tomato for the mold to have to eat through".

Explanations referring to the *ratio* between SA and V were relatively few (n=8) in pre-survey responses, but became more frequent (n=28) in post-survey responses, suggesting the instruction possibly helped students to familiarize themselves with this concept. Here are some example responses:

- When answering Q4, Pamela said in the pre-survey: "Higher surface area: mole ratio means a faster reaction", and changed her response in the post-survey: "SA/volume ratio is greater for nano, not just SA by itself".

- For Q6, Paige wrote in the pre-survey: "e^- will be transformed in and out of the cristae, by folding it increasing the rate of reaction – more surface. Probably the same idea as villi in intestine", and in the post-survey: "Increasing surface area to volume ratio – more reactivity".
- In responding to Q7, Marisa said in the pre-survey: "Surface area causes most of the other material to be touching at one time, making the reactions go faster", and in the post-survey: "Again powder has a larger surface area/volume than the pellets".
- When answering Q10, Keith wrote in the pre-survey: "The many cherry tomatoes have a larger, collective surface area than the beefeater tomatoes and would therefore be exposed to more of the mold spray than the beefeater tomatoes", and in the post-survey very briefly: "Surface area to volume ratio".

Some responses pertaining to SA went beyond SA itself, and mentioned that property-change was due to increased exposure of the molecules or atoms on the surface (n=30), or even the different characteristics of these molecules/atoms (n=3). Zed for example said the following for Q7: "In the pellet, the inner atoms are trapped for some time. The powdered atoms are all available from the beginning." Kyle answered in Q9: "They *(the choices)* are all saying the same thing – more surface area = more exposed molecules, which have higher energy and need less to be broken off." Adam gave similar answers to Q9: "Surface molecules are bonded to less sugar molecules, so their surface energy is higher (allowing for more reactions)."

In summary, the survey responses confirmed what we found in the initial study – many students' understanding of SA/V tends to focus only on the surface component of SA/V, and the idea that what matters is not surface area alone but the ratio between surface area and volume is grasped by surprisingly few students. In addition, few students seemed to understand the mechanism between SA/V ratio (or SA) and property change – the different characteristics or behaviours of surface molecules/atoms. To them, size-dependent properties occur because of more surface area or surface molecule/atom exposure (e.g. to reactants). While this could be considered a reasonable explanation for higher reactivity or solubility when an object is cut into small pieces, it does not help explain why properties such as melting point change at the nanoscale. Given that surface molecule characteristics were only mentioned 3 times in the survey responses, we believe that many students' understanding of SA/V is situated in limited property contexts (e.g. reactivity, solubility) only.

Discussion

The findings reported here, together with what was suggested in the initial study (Light et al., 2008) led to the development of a preliminary typology of SA/V conceptions (Figure 2). Guided by the Variation Theory (Marton & Booth, 1997), the typology includes three types and seven sub-types of conceptions that are characterized and differentiated by seven aspects of variation. The three types describe a progression of conceptions from the least to the most sophisticated – students who do not see connections between SA/V and property change (Type 3), those who only connect separate component(s) of SA/V to property change (Type 2), and those who have an integrated understanding of the relationship between the two (Type 1). For sub-types 3a and 3b, while both do not connect SA/V with property change at the nanoscale, conception 3a recognizes that properties change as size decreases whereas 3b does not. The three sub-types within Type 2 all see the connection between SA/V and property change, but differ in the components of SA/V that are salient in their conception — conception 2c focuses on surface area exposure only; conception 2b is concerned with surface area molecule exposure in addition to surface area; and conception 2a connects both surface area and volume to property change, but separately so. Within Type 1, which understands that it is the ratio between SA and V that explains property change, conception 1a is more sophisticated than conception 1b in that it understands the mechanism behind the SA/V-property change link, i.e. the different characteristics of surface and bulk molecules/atoms. The aspect of variation on which a particular sub-type distinguishes from the others is listed correspondingly at the bottom of the typology, and the specific manner it differs from the other conceptions is included in the conception description. These types and sub-types constitute a cognitive map or outcome space describing and contrasting student conceptions, and the aspects of variation could serve as a way of diagnosing what is missing from a particular type of conception.

It is interesting that while more references to SA/V were seen in post-survey responses, no increase regarding SA molecules/atoms or their properties occurred. This suggests an improvement of conception from Type 2 to Type 1b, but not the most sophisticated type (Type 1a). The practical implication of this observation is that the mechanism underlying SA/V and property change needs to be explicitly addressed during instruction, in order to avoid the pitfall mentioned above, namely that students' understanding was limited to the context of certain properties only.

Conceptions of Surface Area to Volume Ratio with Respect to Property Change at the Nanoscale: A Preliminary Typology of Undergraduate Student Understanding							
Type of conception	**Type 3** Surface Area and Volume (SA/V) are understood as separate from property change at the nanoscale		**Type 2** Surface Area and Volume (SA/V) are understood as related to property change at the nanoscale, but separately so			**Type I** Surface Area and Volume (SA/V) are understood as related to property change in an integrated ratio at the nanoscale	
Sub-type	3b	3a	2c	2b	2a	Ib	Ia
Description	• Understands SA/V as a simple mathematical ratio unrelated to property change at nanoscale • No link of nanoscale to property change	• Understands that there is a relationship between nanoscale and property change • No link of property change to Surface area (SA) or Volume (V)	• Understands that proper change at nanoscale is related to change in SA • No link of property change to SA molecule exposure	• Understands that property change at nanoscale is related to increased number of exposed molecules at the surface • No link of property change to V	• Understands that property change at nanoscale is related to V in addition to SA, but sees the two relationships as separate • No link of property change to SA/V ratio	• Understands that property change at nanoscale is related to an integrated *ratio* of SA/V • No link of property change to different characteristics of surface (vs. Bulk) molecules	• Understands that property change is related to the different characteristics of surface (vs. Bulk) molecules
	Descriptions are hierarchically inclusive →						
Aspects of variation	SA/V as a simple ratio	Related to property change	Related to SA	Related to surface molecule exposure	Related to SA and V	Related to SA/V ratio	Related to surface molecule characteristics

Figure 2. Typology of student conceptions of SA/V

The application of the CCD framework in our study is quite successful. Designing assessment items based on conception unpacking proved to be a useful approach, but perhaps what is more important is that the iterative nature of CCD allows us to further unpack student conceptions while simultaneously developing and testing assessment items. By focusing on actual student responses rather than experts' opinions, we believe our typology provides a more intimate and perhaps accurate portrait of student conceptions.

It should be pointed out that our assessment items were administered with only 16 students who share similar profiles (e.g. engineering major, freshmen), thus the generalizability of our findings to other populations still needs to be confirmed. We are currently planning studies involving student groups of more or less advanced levels, and anticipate that the results collected from them will suggest additional ways of expanding and revising our typology.

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Appendix 1: SA/V items

All items include multiple choices and a follow-up question for reasoning. However, due to space limitation, the multiple choices and follow-up question are not shown here. This information can be obtained by contacting the authors.

Q4. Suppose you are given two cubes of the same material – The edges of Cube A are 10 cm in length, and the edges of Cube B are 10 nm in length. Both cubes react with oxygen. You are asked to test the rate of reaction between these cubes and oxygen. What kind of test result would you expect, and why?

Q6. Mitochondria are the energy organelles of the cell. They are primarily responsible for the extraction of energy from the nutrients in food with oxygen and transfer it for use in other cellular activities. The proteins required for this transfer are embedded in the folds called *cristae* within the organelle. (See diagram). Which one of the following best accounts for the highly folded nature of the *cristae*? *(Diagram omitted due to space limit.)*

Q7. A student conducts two experiments reacting hydrochloric acid (HCl) with metallic zinc (Zn) to produce zinc chloride ($ZnCl_2$) and hydrogen gas (H_2): $Zn_{(s)} + 2HCl_{(aq)} \rightarrow ZnCl_{2(aq)} + H_{2(g)}$

Both experiments used 25 mL of 6 M HCl.

- In Experiment *I*, a 5.0 g *pellet* of zinc was used.
- In Experiment *II*, 5.0 g of *powdered* zinc was used.

Experiment II using the *powdered* zinc was found to go faster to completion than Experiment I. The reason for this result would best be described as___________________.

Q9. You may know from experience that it takes longer to dissolve a cube of sugar in water than the same amount of powdered sugar. Circle all of the following answers that you think provide the reason for this difference?

Q10. You have two 1-lb bags of tomatoes, one made up of several large beefeater tomatoes, the other made up of many small cherry tomatoes. You spread the tomatoes out on a flat table and spray them with mold, and then return them to their respective bags. Which bag of tomatoes is likely to totally rot more quickly?

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